

# Robust design of railway vehicle suspension using a process capability index<sup>†</sup>

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## Abstract

Robust design for the primary suspension of a railway vehicle was performed according to the optimization of 10 dynamic responses representing driving safety and ride comfort, in which response surface models (RSMs) from the design of experiments (DOEs) were applied. To evaluate the probabilistic feasibility of robustness, an intensive computational process is mandatory. In the present study, the authors utilized the first-order Taylor series expansion to reduce the computational burden associated with a probabilistic feasibility evaluation, thus easily obtaining both an individual mean and variance of constraints. To overcome the difficulty of optimizing the mean and probabilistic variances for the 10 dynamic responses, a process capability index ( $C_{pk}$ ) was introduced, which shows the mean value and scattering of the product quality to a certain extent and normalizes the objective functions irrespective of varying dimensions. Consequently, the robust design to optimize the 10 dynamic responses minimized the  $C_{pk}$  subjected to the constraint of  $C_{pk} \geq 2$ , which satisfied  $6\sigma$ . The proposed method improved the  $C_{pk}$  that violated the constraints obtained by the RSMs from DOEs and minimized the variance of the  $C_{pk}$ .

**Keywords:** Design of experiments; Process capability index; Railway vehicle suspension; Robust design; Six sigma

## 1. Introduction

Robust design optimization has increased in practical applications as design problems are increasingly analyzed from a robust point of view [1, 2]. One way to achieve robust design optimization is to employ statistical techniques based on design-of-experiment (DOE) and response-surface-model (RSM) approaches. These techniques construct algebraic regression approximations of object functions and constraints, usually through second-order polynomial regression equations. Algebraic regression approximations such as RSMs are estimates of responses at generated data points that are carefully distributed throughout a design space with the aid of a DOE. A large variety of experimental designs, for example, central composite designs, D-optimal designs, full factorial or fractional factorial designs, and orthogonal arrays, are widely used.

In a robust design of the primary suspension of a railway vehicle, it is important to not only achieve the robust design objectives but also to maintain the robustness of design feasibility when subjected to variations, also called uncertainties. To evaluate the probabilistic feasibility of robustness, an in-

tensive computational process is mandatory. The authors utilized the first-order Taylor series expansion [3] to reduce the computational burden associated with probabilistic feasibility evaluation, thus easily estimating both an individual mean and the variance of constraints. To overcome the difficulty of optimizing the mean and the probabilistic variances of various dynamic responses of a railway vehicle suspension with exact probabilistic variances, a  $C_{pk}$  [4] was introduced, which shows the mean value and scattering of product quality to a certain extent. The present study proposes a procedure for the robust design of a railway vehicle suspension that uses a  $6\sigma$  robust design and considers the  $C_{pk}$  obtained from its dynamic responses.

## 2. $6\sigma$ robust design

### 2.1 Robust design using a first-order Taylor expansion

The analysis and optimization of a railway vehicle suspension was carried out based on RSMs constructed from DOEs [5]. However, the number of DOEs increases exponentially according to the number of design variables. To reduce the number of DOEs to a more practical level, fractional factorial designs are recommended at the first analysis step, which is called screening, to explore the main effects and interactions of design variables. Orthogonal array is the smallest fractional

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factorial analysis design with one main effect among the factors. Taguchi’s orthogonal array is commonly used in the industry and is well-balanced, with a standard design to examine a main effect and interactions of design variables. It is easy to use and employs a linear graph and a triangular table.

Efficient RSMs were created to replace the VAMPIRE [6], which enables the calculation of the dynamic responses of a railway vehicle suspension through an approximation technique that uses the D-optimal design [7] for optimization as follows:

$$Y = X^T \beta + \varepsilon = X^T [(X^T X)^{-1} X^T Y_{obj}], \tag{1}$$

where  $X$ ,  $Y_{obj}$ , and  $Y$  denote the design variables, responses from the VAMPIRE, and predicted responses, respectively. Coefficients of the RSMs  $\beta$  were evaluated using a least square fitting. A robust design is intended to improve the quality of a product or process by minimizing the effects of variation without eliminating the causes of the variation while simultaneously striving to achieve performance targets. The design problem was considered for a given range of  $x$  and  $z$  using an objective function  $f(x, z)$  as follows:

Minimize:  $\mu_f, \sigma_f$   
 Subject to:

$$g_j(x, z) + \sqrt{\sum_{i=1}^n \left( \frac{\partial f}{\partial x_i} \sigma_{x_i} \right)^2 + \sum_{i=1}^1 \left( \frac{\partial f}{\partial z_i} \sigma_{z_i} \right)^2} \leq 0 \tag{2}$$

$x_L + \Delta x \leq x \leq x_U - \Delta x,$

where  $x$  and  $z$  represent the design variables and uncontrollable parameters, and  $\mu_f$  and  $\sigma_f$  are the mean and standard deviation of the objective function  $f(x, z)$ , respectively. Eq. (2), which is most frequently used to approximate  $\sigma_f$ , is predicted by the probabilistic approximation of a first-order Taylor expansion [3].

**2.2 Robust design using  $C_{pk}$**

The present study induced the capability index  $C_{pk}$  [4], which shows the mean value and variance of product quality to a certain extent and normalizes the objective functions irrespective of varying scales.  $C_{pk}$  is defined as follows:

$$C_{pk} = \text{Min} \left( \frac{USL - \mu_f}{3\sigma}, \frac{\mu_f - LSL}{3\sigma} \right), \tag{3}$$

where  $USL$  and  $LSL$  are the upper and lower specification limits, respectively. When the  $C_{pk}$  is greater than two, the process capability exceeds the  $6\sigma$  level, which is why it is called a  $6\sigma$  robust design [8]. Consequently, the design for a robust optimization of the dynamic responses of a railway vehicle suspension is proposed as follows:

Maximize:  $C_{pk}$   
 Subject to:  $C_{pk} \geq 2$  (4)  
 $x_L + \sigma_x \leq x \leq x_U - \sigma_x$

**3. Modeling of a railway vehicle suspension**

A railway vehicle is composed of six members, as shown in Fig. 1: one power motor (24), one motorized trailer (1), three trailers (2, 3, and 4) and another motorized trailer (5). There are eight bogies: two power motor bogies (PMB, 25 and 26), one motorized trailer bogie (MTB, 6), four articulated trailer bogies (ATB, 7, 8, 9, and 10), and another motorized trailer bogie (MTB, 11). The present study addressed a PMB and an ATB using three design variables, namely,  $k_x, k_y$  and  $k_z$ , which represent the elastic coefficients in the primary suspension. The dynamic responses of a railway vehicle suspension calculated using the VAMPIRE were composed of 10 ride (comfort) parameters, 24 derailment quotients, and 24 unloading ratios. Among the 58 responses, the present considered 10 dynamic responses with significant influence on either a curved or a straight track with and without a wind load, as shown in Table 1.

Table 1. Ten dynamic responses of a railway vehicle suspension.

| Condition                                | Object functions | Performance index           |
|--|------------------|-----------------------------|
| Wind on straight track (transient state) | Imlw1            | Unloading on left wheel 1   |
|  | Imruw1           | Unloading on right wheel 1  |
|  | Imldw1           | Derailment on left wheel 1  |
|  | Imrdw1           | Derailment on right wheel 1 |
| Curved track (quasi-static state)        | Culdw1           | Derailment on left wheel 1  |
|  | Culdw2           | Derailment on left wheel 2  |
|  | Curuw1           | Unloading on right wheel 1  |
|  | Curuw2           | Unloading on right wheel 2  |
| Straight track                           | Rilat            | Ride comfort in Y direction |
|  | Rivert           | Ride comfort in Z direction |

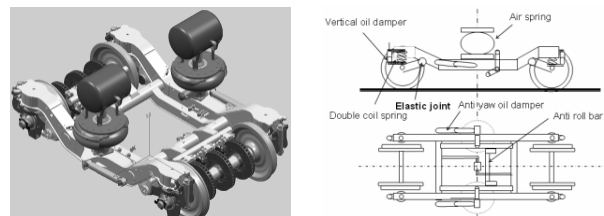
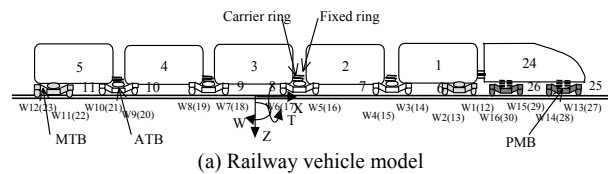


Fig. 1. Model of a railway vehicle and its suspension system.

Table 2. Accuracy of the RSMs.

| Object functions | R <sup>2*</sup> | R <sup>2**</sup> <sub>adj</sub> |
|------------------|-----------------|---------------------------------|
| Imluw1           | 0.97            | 0.97                            |
| Imruw1           | 0.92            | 0.91                            |
| Imldw1           | 0.98            | 0.98                            |
| Imrdw1           | 0.96            | 0.96                            |
| Culdw1           | 0.95            | 0.95                            |
| Culdw2           | 0.98            | 0.98                            |
| Curuw1           | 0.99            | 0.99                            |
| Curuw2           | 0.93            | 0.93                            |
| Rilat            | 0.93            | 0.92                            |
| Rivert           | 0.99            | 0.98                            |

\* R<sup>2</sup>: coefficient of determination  
 \*\* R<sup>2</sup><sub>adj</sub>: adjusted coefficient of determination

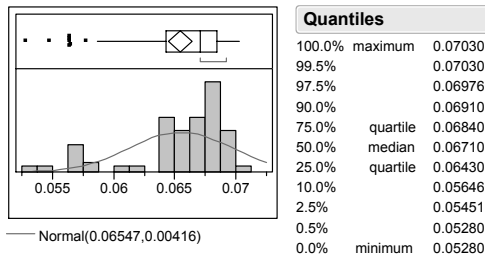
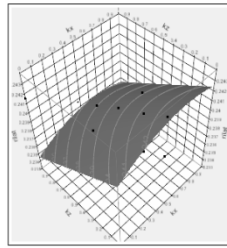


Fig. 2. USL and standard deviation of Culdw2.

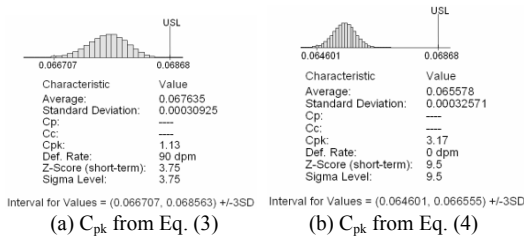


Fig. 3. C<sub>pk</sub> of Culdw2 calculated from Eq. (2) and Eq. (4).

## 4. Results

### 4.1 Robust design from RSMs

Seventy-five DOEs, L<sub>75</sub> (5<sup>3</sup>), were carried out according to the D-optimal design for five levels of optimization. The well-known indices representing the accuracy of the obtained RSMs are a coefficient of determination and its adjustment [9]. Table 2 shows the values obtained for the RSMs of each performance index. All values exceeded 0.9; therefore, the obtained RSMs represent more than 90% of the true data.

In their current design state, the C<sub>pk</sub> of the 10 object functions were estimated, and a 75% quartile of distribution was determined as the USL, with 5% of the mean selected as the standard deviation according to the given design specifications. Fig. 2 shows an example of the distribution for Culdw2. The current C<sub>pk</sub> was simulated by performing the Monte Carlo simulation 10,000 times. Fig. 3 (a) shows an example of the C<sub>pk</sub> for Culdw2 at 1.13 (3.75σ). All values of C<sub>pk</sub> (○) calculated from the RSMs using Eq. (3) are plotted in Fig. 4, and three of these C<sub>pk</sub>, namely, Imldw1, Culdw2, and Rilat, violated the constraints of Eq. (4).

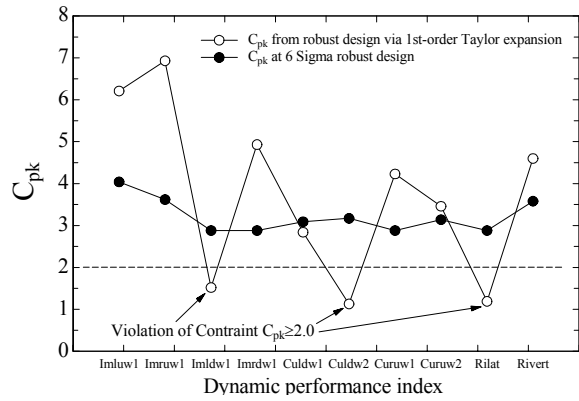


Fig. 4. Comparison of C<sub>pk</sub> calculated from the robust design through the first-order Taylor expansion and 6σ robust design.

### 4.2 6σ robust design

The 6σ robust design was carried out using Eq. (4). Fig. 3 (b) shows that the C<sub>pk</sub> of Culdw2 increases to 3.17 (9.6σ). All C<sub>pk</sub> (●) obtained from performing the Monte Carlo simulation are plotted in Fig. 4. The three violating C<sub>pk</sub>, Imldw1, Culdw2, and Rilat, were optimized to 2.88, 3.17, and 2.88, respectively, to satisfy the constraint, C<sub>pk</sub> ≥ 2. In addition, the variance of C<sub>pk</sub> improved significantly.

In practice, when probabilistic variances are unpredictable, it cannot be determined whether constraints are violated or not. The 6σ robust design, consequently, allows for both evaluation of the probabilistic feasibility of constraints and minimization of constraint violations.

## 5. Conclusions

The present study proposed a procedure for the robust design of a railway vehicle suspension that uses a 6σ robust design and considers the C<sub>pk</sub> obtained from its dynamic responses. It is possible that the C<sub>pk</sub> can overcome the difficulty of the trade-off between the mean and the probabilistic variances for the 10 dynamic responses of a railway vehicle suspension. In addition, the objective functions can be normalized within a single index, C<sub>pk</sub>, to solve the scale difficulty in optimization, irrespective of various dimensions. The proposed 6σ design method was confirmed by Monte Carlo simulation, in which the optimized C<sub>pk</sub> satisfied the subjected constraint at greater than 6σ level. Its variance also improved significantly.

Consequently, the C<sub>pk</sub> enabled the normalization of different object functions and constraints and acted as a communication channel for product quality of manufacturing due to its capability in representing the 6σ index.

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